to accumulate additional data derived from the examination of other parallel cases, such as the ternary alloys obtained by adding tin to the immiscible pairs of metals, zinc and bismuth, aluminium and lead, and aluminium and bismuth; or by similarly employing other metals instead of tin. Nothing abnormal appears to characterise the solubility curves of zinc in lead-tin and of lead in zinc-tin; in each case the amount of one metal dissolved by the other increases as the quantity of tin present increases, in such a way that the curves are somewhat concave upwards.

IV. " The Diurnal Variation of Terrestrial Magnetism." By ARTHUR SCHUSTER, F.R.S., Professor of Physics, with an Appendix by H. LAMB, F.R.S., Professor of Mathematics, Owens College, Manchester.   Received March 20, 1889.

(Abstract.)

In the year 1839 Gauss published his celebrated Memoir on Terrestrial Magnetism, in which the potential on the earth's surface was calculated to twenty-four terms of a series of surface harmonics. It was proved in this memoir that if the horizontal components of magnetic force were known all over the earth the surface potential could be derived without the help of the vertical forces, and it is well known now how these latter can be used to separate the terms of the potential which depend on internal from those which depend on external sources. Nevertheless, Gauss made use of the vertical forces in his calculations of the surface potential, in order to ensure a greater degree of accuracy. He assumed for this purpose that magnetic matter was distributed through the interior of the earth, and mentions the fair agreement between calculated and observed facts as a justification of his assumption. In the latter part of the memoir it was suggested that the same method should be employed in the investigation of the regular and secular variations.

The use of harmonic analysis to separate internal from external causes has never been put to a practical test, but it seems to me to be specially well adapted to inquiries on the causes of the periodic oscillations of the magnetic needle.

If the magnetic effects can be fairly represented by a single term in the series of harmonics as far as the horizontal forces are concerned, there should be no doubt as to the location of the disturbing cause, for the vertical force should be in the opposite direction if the origin is outside from what it should be if the origin is inside the earth. As the expression for the potential contains in one case the distance from the earth's centre in the numerator, in the other case in the denominator, and as the vertical force depends on the diffe-

rential coefficient with regard to the distance from the earth to the centre, each single term in the series is of opposite sign according to the location of the cause; but what is true for each single term need not be true for the sum of the series. By a curious combination of terms the vertical forces might possibly be of the same sign on whichever of the two hypotheses it is calculated. In any case, however, the differences between the two results will be of the same order of magnitude as the vertical force itself. If it were then a question simply of deciding whether the cause is outside or inside, without taking into account a possible combination of both causes, the result should not be doubtful, even if we have only an approximate knowledge of the vertical forces.

Two years ago I showed that the leading features of the horizontal components for diurnal variation could be approximately represented by the surface harmonic of the second degree and first type, and that the vertical variation agreed in direction and phase with the calculation on the assumption that the seat of the force is outside the earth. The agreement seemed to me to be sufficiently good to justify the conclusion that the greater part of the variation is due to causes outside the earth's surface. Nevertheless, it seemed advisable to enter more fully into the matter, as in the first approximate treatment of the subject a number of important questions had to be left untouched. I now publish the results of an investigation which has been carried out, as far as the observations at my disposal have allowed me to do. My original conclusions have been fully confirmed, and some further information has been obtained, which I believe to be of importance.

I have made use of the observations taken at Bombay, Lisbon, Greenwich, and St. Petersburg. The horizontal components of the diurnal variation during the year 1870 were in the first place reduced to the same system of coordinates and to the same units. If we remember that experience has shown the diurnal variation to be very nearly the same for places in the same latitude, except near the magnetic pole, and also that it is symmetrical north and south of the equator, we may for a given time of day assume the horizontal components known over eight circles of latitude, four of which are north and four south of the equator. If we chose the period of the year for which the reduction is made to be that corresponding to the summer months in the northern hemisphere, we must take the variation in the southern hemisphere to be the same as that found during the winter months north of the equator. This was done in one part of the inquiry; in the other the mean of the whole year was taken, and in that case the same values hold north and south of the equator, with the same sign for the force towards geographical north, and opposite sign for the force towards the geographical west.

From the horizontal components the potential was calculated in terms of a series of surface harmonics. It was found that in order to represent both the summer and the winter effect with sufficient accuracy thirty-eight terms were necessary. In this calculation the vertical forces were not made use of at all.

Unfortunately we do not possess complete records for the vertical force variation during the year 1870, except at Lisbon; but the type of that force is very nearly the same from year to year, varying only slightly in amplitude. It is shown that, as far as the conclusions drawn in the paper are concerned, an accurate knowledge of the amplitude of the vertical force is not required. I have chosen for comparison the vertical force of Bombay in the year 1873, and for Greenwich in 1882. As regards St. Petersburg, vertical force records exist for 1870, but they have not been corrected for temperature variations of the magnet, and are therefore of doubtful importance. I have therefore used the St. Petersburg observations for 1878, in addition to those for 1870.

From the potential, as calculated from the horizontal components, we can deduce the vertical force, either on the assumption that the variation is due to an outside cause, or that it is due to an inside cause; and compare the vertical forces thus found with the vertical forces as actually observed.

If we put both into the form

$$r_n \cos n \ (t - t_n),$$

we can obtain an idea of the agreement as regards amplitude and phase for each harmonic term. The following tables give the results for $n = 1$ and $n = 2$, that is, for the diurnal and the semi-diurnal variation.

Table I.

Observed and calculated Values of the Coefficients t_1 and t_2 of Vertical Force, when expressed in the form $r_1 \cos (t - t_1) + r_2 \cos 2 (t - t_2)$, on the supposition that the Disturbing Force is *inside* the Earth.

	$t_1.$			$t_2.$		
	Calc.	Obs.	Diff.	Calc.	Obs.	Diff.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
Bombay	23 02	11 13	+11 49	9 55	4 23	+5 32
Lisbon..............	22 35	10 40	+11 58	11 42	5 50	+5 52
Greenwich	22 06	8 42	−11 57	11 32	5 56	+5 36
St. Petersburg, 1870.	21 16	3 10	− 5 54	10 48	7 05	+3 43
,, 1878.	..	7 05	− 9 49	..	6 12	+4 36

Table II.

Observed and calculated Values of the Coefficients t and t_2 when expressed in the form, $r_1 \cos(t-t_1) + r_2 \cos 2(t-t_2)$, on the supposition that the disturbing force is *outside* the Earth.

	t_1.			t_2.		
	Calc.	Obs.	Diff.	Calc.	Obs.	Diff.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
Bombay	11 10	11 13	−0 03	3 47	4 23	−0 36
Lisbon.............	10 37	10 40	−0 03	5 46	5 50	−0 04
Greenwich	10 03	8 42	+1 21	5 38	5 56	−0 18
St. Petersburg, 1870.	8 52	3 10	+5 42	4 38	7 05	−2 27
„ 1878.	..	7 05	−1 47	..	6 12	−1 34

Table III.

Observed and calculated Values of r_1 and r_2 in the Expression $r_1 \cos(t-t_1) + r_2 \cos_2(t-t_2)$ for Vertical Force.

	r_1.			r_2.		
	Calculated from inside.	Calculated from outside.	Observed.	Calculated from inside.	Calculated from outside.	Observed.
Bombay	236	144	43	171	132	35
Lisbon.............	491	346	176	333	277	153
Greenwich	398	269	65	143	112	51
St. Petersburg, 1870.	235	142	169	77	53	71
„ 1878.	..	..	30	..	..	24

In Table I the comparison of the observed phases is made with the values calculated on the assumption that the disturbing force is inside the earth. In Table II the same comparison is made on the alternative hypothesis. There is complete disagreement in Table I between the observed and calculated values, and nearly complete agreement in Table II. It is seen how both at Lisbon and Bombay the time of maximum displacement agrees within three minutes of time for the diurnal variation, and at Lisbon within four minutes of time also for the semi-diurnal variation. Considering that Lisbon is the most important station, not only on account of its geographical position, but also because the observed vertical forces apply to the

same year as the calculated ones, the result is strikingly in favour of the outside force. The results for Greenwich argue in the same direction. As regards St. Petersburg, the results for 1870 neither agree with one nor with the other hypothesis, and it has already been mentioned that the observations for 1870 are doubtful, but the results for 1878 agree well with the hypothesis of an outside disturbing force.

Table III gives the comparison for amplitude. It is seen that the observed amplitudes are throughout smaller than the calculated ones. If curves are drawn representing the results of Tables I, II, III, it is clearly seen how well the calculated vertical forces agree with the observed ones as regards phase, if we assume the cause of the variation to be outside.

If we then take it as proved that the primary cause of this variation comes to us from outside the earth's surface, we are led to consider that a varying magnetic potential must cause induced currents within the earth, if that body is a sufficiently good conductor. These induced currents might be the cause of the apparent reduction in amplitude. As my colleague, Professor Lamb, has given considerable attention to the problem of currents in a conducting sphere, I consulted him, and he gave me the formulæ by means of which the induced currents can be calculated. His investigation is given in an appendix to the paper. The result is very interesting. If the earth is treated as a conducting sphere, the observed reduction in amplitude is accounted for, but that reduction should be accompanied by a change of phase which is not given by observation. We can reconcile all facts if we assume, as suggested by Professor Lamb, the average conductivity of the outer layers of the earth to be very small, so that the reduction in amplitude is chiefly due to currents induced in the inner layers. If the conductivity inside is sufficiently large, a considerable reduction in amplitude would not be accompanied by a sensible change of phase. We have arrived, therefore, at the following result :—

The vertical forces of the diurnal variation can be accounted for if we assume an outside cause of the variation, which induces currents in the earth, and if the earth's conductivity is greater in the lower strata than near the surface.

Professor Balfour Stewart's suggestion that convection currents in the atmosphere moving across the lines of the earth's magnetic forces are the causes of the daily variation, gains much in probability by this investigation. If the daily variation of the barometer is accompanied by a horizontal current in the atmosphere similar to the tangential motion in waves propagated in shallow canals, and if the conductivity of the air is sufficiently good, the effects on our magnetic needles would be very similar to those actually observed. The difficulty as to the conductivity of the air is partly met by the author's

investigation of the behaviour of gases through which electric discharges are passing.

It will be interesting to follow out the investigation, especially with a view of examining the influence of sun-spot variation. The question of magnetic disturbances is more complicated, but as magnetical observatories are being established in many countries, the time may not be far distant when we shall be able to bring the irregular disturbances within the reach of calculation.

In order to facilitate the necessarily long computations, the author makes an appeal to the heads of magnetic observatories to reduce the regular variation according to the method adopted by Wild at St. Petersburg, or that in use at Greenwich, the two being nearly identical. The variations should also be reduced to the geographical coordinates, instead of to magnetic coordinates.

The author acknowledges the help he has received from Mr. William Ellis in some of the reductions; he has also to thank his assistant, Mr. A. Stanton, for much labour bestowed on making and checking numerical calculations.

V. " On the Conditions for effective Scour in Drain-pipes of Circular Section." By HENRY HENNESSY, F.R.S., Professor of Applied Mathematics and Mechanism in the Royal College of Science for Ireland. Received March 1, 1889.

Presents, March 28, 1889.

Transactions.

Edinburgh :—Royal Society.   Proceedings.   Vol. XVI.   No. 129.
    8vo. *Edinburgh* 1888.   The Society.
Essex Field Club :—The Essex Naturalist.   1888.   Nos. 11–12. 8vo.
    Buckhurst Hill.   The Club.
Kew :—Royal Gardens.   Bulletin of Miscellaneous Information.
    No. 27. 8vo. *London* 1889.   The Director.
Pisa :—Società Toscana di Scienze Naturali.   Processi-Verbali.
    1888. Novembre. 8vo. [*Pisa* 1889.]   The Society.
Stockholm :—Kongl. Svenska Vetenskaps-Akademie.   Bihang till
    Handlingar (Afd. 1). Bd. XIII; Ditto (Afd. 2). Bd. XIII;
    Ditto (Afd. 3). Bd. XIII. 8vo. *Stockholm* 1888; Öfversigt.
    Årg. XLVI. No. 1. 8vo. *Stockholm* 1889.   The Academy.
Sydney :—Linnean Society of New South Wales.   Proceedings.
    Vol. III. Part 3. 8vo. *Sydney* 1888.   The Society.